

# Lepton flavor violating decays of $\mu$ and $\tau$ leptons in a gauge group

$$SU(2)_L \times SU(2)_R \times SU(2)_Y$$

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The electroweak unification group  $SU(2)_L \times SU(2)_R \times SU(2)_Y$  is proposed for the charged lepton flavor violating decays of the muon ( $\mu$ ) and tau ( $\tau$ ) leptons. The group  $SU(2)_Y$  is in the lepton space. The left-handed leptons and anti-leptons are assigned to the fundamental representation  $(2, 2, \bar{2})$  of the semi-simple group. The gauge group  $SU(2)_Y$  is spontaneously broken to  $U(1)_{Y_1}$ , where  $Y_1 = -L = \pm 1$  is the hypercharge, by introducing a scalar multiplet  $\Sigma$  which belongs to the triplet representation 3 of the  $SU(2)_Y$  and is singlet under  $SU(2)_L \times SU(2)_R$ . At this stage charged vector bosons  $Y^\pm$  of  $SU(2)_Y$  which mediate the lepton flavor violating decays acquire masses and are decoupled with one Higgs scalar  $H_\Sigma^0$ . The residual group  $SU(2)_L \times SU(2)_R \times U(1)_{Y_1}$  has all the features of the left-right electroweak unification group extensively studied in the literature. The probability for lepton flavor violating decays is  $\left(\frac{\sin^2 \theta_W}{1-2\sin^2 \theta_W}\right)^2 \left(\frac{m_{W_L}}{m_Y}\right)^4$ .

## I. INTRODUCTION

In the standard model, lepton number and baryon number are conserved, i.e.  $\Delta L = 0$  and  $\Delta B = 0$ . Bounds on the lifetimes of electron and proton are

$$\tau_e > 4.6 \times 10^{26} \text{ years}, \quad \tau_p > 10^{31} \text{ years}. \quad (1)$$

For the leptons, there is another conservation law, viz the lepton number for each generation is conserved. No process with  $\Delta L_e \neq 0$ ,  $\Delta L_\mu \neq 0$  and  $\Delta L_\tau \neq 0$  is allowed. The left-handed current

for weak decays in the standard model is

$$J_{\text{lepton}}^{+\mu} = (\bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau) \gamma^\mu (1 - \gamma^5) \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} = [\bar{\nu}_e \gamma^\mu (1 - \gamma^5) e + \dots] \quad (2)$$

$$J_{\text{quark}}^{+\mu} = (\bar{u}, \bar{c}, \bar{t}) \gamma^\mu (1 - \gamma^5) V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix} = [\bar{u} \gamma^\mu (1 - \gamma^5) (V_{ud} d + V_{us} s + V_{ub} b) + \dots] \quad (3)$$

where  $V_{CKM}$  is the Cabibbo–Kobayashi–Maskawa (CKM) matrix. Unlike the quark-sector, where the flavor changing weak decays are allowed, in the lepton-sector no lepton flavor changing decays are allowed.

In the standard model, the neutrinos are only left-handed and hence are massless. For massless neutrinos, no linkage between three generations. Mixing between neutrinos is possible if all the neutrinos are not massless. Mixing requires that mass eigenstates  $\nu_i$ , ( $i = 1, 2, 3$ ) are different from flavor eigenstates. In this case the oscillations are possible. In the neutrino oscillations,  $\nu_e \rightarrow \nu_\mu$  and  $\nu_\mu \rightarrow \nu_\tau$  have been experimentally observed [1, 2] .

One has to go beyond the standard model to explore the charged lepton flavor violating decays. In this context, the electroweak unification gauge group  $SU(2)_L \times SU(2)_R \times SU(2)_Y$  is proposed, the group  $SU(2)_Y$  is in the leptonic space. The concept of isospin in the leptonic space was first introduced in 1975 [3]. The left-handed leptons and antileptons are assigned to the fundamental representation  $(2, 2, \bar{2})$  of the gauge group:

$$\Psi = \begin{array}{c} \xleftarrow{SU(2)_Y} \\ \left| \begin{pmatrix} \nu_n & e_m^c \\ e_n & -N_m^c \end{pmatrix} \right| \\ \downarrow SU(2)_L \quad \downarrow SU(2)_R \end{array} \quad (4)$$

where  $n$  and  $m$  are the flavor indices and the superscript  $c$  denotes the charge-conjugation. The multiplets  $(\nu_n, e_n)_L^T$  and  $(e_m^c, -N_m^c)_L^T$  belong to the fundamental representations of  $SU(2)_L$  and  $SU(2)_R$ , respectively, whereas two doublets  $(\nu_n, e_m^c)_L$  and  $(e_n, -N_m^c)_L$  belong to the representation  $\bar{2}$  of  $SU(2)_Y$ . There are three sets of vector bosons  $(W_L^\pm, W_{L\mu}^0)$ ,  $(W_{R\mu}^\pm, W_{R\mu}^0)$  and  $(Y_\mu^\pm, Y_\mu^0)$  belonging to the adjoint representation of each  $SU(2)$  gauge group. Out of six charged vector bosons, the four  $W_{L\mu}^\pm$ ,  $W_{R\mu}^\pm$  are coupled to the left-handed and right-handed weak currents  $J_L^{\pm\mu}$  and  $J_R^{\pm\mu}$  respectively. The remaining two charged vector bosons  $Y_\mu^\pm$  are coupled to the lepton flavor violating currents  $J_Y^{\pm\mu}$ . The gauge bosons  $Y_\mu^\pm$  mediate the lepton number  $L_e$ ,  $L_\mu$  and  $L_\tau$  violating processes. The linear combinations of three neutral vector bosons  $W_{L\mu}^0$ ,  $W_{R\mu}^0$  and  $Y_\mu^0$  give three physical vector

bosons  $A_\mu$ ,  $Z_\mu$  and  $Z'_\mu$  coupled is the electromagnetic current  $J^{em\mu}$ , the weak neutral currents  $J^{Z\mu}$  and the  $J^{Z'\mu}$ , respectively.

Note that for the gauge group  $SU(2)$ , the representations 2 and  $\bar{2}$  are equivalent and it is anomaly free unlike  $SU(N)$  for  $N > 2$  gauge groups which are not anomaly free. Hence, the gauge group  $SU(2)_L \times SU(2)_R \times SU(2)_Y$  is anomaly free.

## II. INTERACTION LAGRANGIAN

The gauge invariant Lagrangian for the fundamental representation  $(2, 2, \bar{2})$  is given by

$$\mathcal{L} = \text{Tr}[i\bar{\Psi}\gamma^\mu\nabla_\mu\Psi], \quad (5)$$

with  $\Psi$  defined in Eq. (4) and

$$\nabla_\mu = \partial_\mu + \frac{i}{2}\tau \cdot W_{L\mu} + \frac{i}{2}\tau \cdot W_{R\mu} - \frac{i}{2}(\tau \cdot Y_\mu)^\dagger, \quad (6)$$

where  $\tau^a$  are the Pauli matrices. Thus the interaction Lagrangian

$$\begin{aligned} \mathcal{L}_{\text{int}} = \frac{i}{2} & \left[ (\bar{\nu}_n, \bar{e}_n)_L \gamma^\mu \begin{pmatrix} W_{L\mu}^0 & \sqrt{2}W_{L\mu}^+ \\ \sqrt{2}W_{L\mu}^- & -W_{L\mu}^0 \end{pmatrix} \begin{pmatrix} \nu_n \\ e_n \end{pmatrix}_L + (\bar{e}_n^c, -\bar{N}_n^c)_L \gamma^\mu \begin{pmatrix} W_{R\mu}^0 & \sqrt{2}W_{R\mu}^+ \\ \sqrt{2}W_{R\mu}^- & -W_{R\mu}^0 \end{pmatrix} \begin{pmatrix} e_n^c \\ -N_n^c \end{pmatrix}_L \right. \\ & \left. - (\bar{\nu}_n, e_m^c)_L \gamma^\mu \begin{pmatrix} Y_\mu^0 & \sqrt{2}Y_\mu^- \\ \sqrt{2}Y_\mu^+ & -Y_\mu^0 \end{pmatrix} \begin{pmatrix} \nu_n \\ e_m^c \end{pmatrix}_L - (\bar{e}_n, -\bar{N}_m^c)_L \gamma^\mu \begin{pmatrix} Y_\mu^0 & \sqrt{2}Y_\mu^- \\ \sqrt{2}Y_\mu^+ & -Y_\mu^0 \end{pmatrix} \begin{pmatrix} e_n \\ -N_m^c \end{pmatrix}_L \right] \quad (7) \end{aligned}$$

From the above equation we can separate the charged and neutral parts of the interaction Lagrangian as

$$\begin{aligned} \mathcal{L}_{\text{int}}^{\text{charge}} = -\frac{1}{2\sqrt{2}} & \left\{ g[\bar{\nu}_n\gamma^\mu(1-\gamma^5)e_nW_{L\mu}^+ + \bar{N}_n\gamma^\mu(1+\gamma^5)e_nW_{R\mu}^+ + \text{h.c.}] \right. \\ & \left. - g_Y[(\bar{e}_m^c\gamma^\mu(1-\gamma^5)\nu_n - \bar{N}_m^c\gamma^\mu(1-\gamma^5)e_n)Y_\mu^+ + \text{h.c.}] \right\} \quad (8) \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{\text{int}}^{\text{neutral}} = -\frac{1}{4} & \left\{ g[(\bar{\nu}_n\gamma^\mu(1-\gamma^5)\nu_n - \bar{e}_n\gamma^\mu(1-\gamma^5)e_n)W_{L\mu}^0 + (\bar{N}_n\gamma^\mu(1+\gamma^5)N_n - \bar{e}_n\gamma^\mu(1+\gamma^5)e_n)W_{R\mu}^0] \right. \\ & \left. - g_Y[\bar{\nu}_n\gamma^\mu(1-\gamma^5)\nu_n + \bar{e}_n\gamma^\mu(1+\gamma^5)e_n + \bar{e}_n\gamma^\mu(1-\gamma^5)e_n + \bar{N}_n\gamma^\mu(1+\gamma^5)N_n]Y_\mu^0 \right\} \quad (9) \end{aligned}$$

In order to express the  $\mathcal{L}_{\text{int}}^{\text{neutral}}$  in terms of physical vector bosons  $A_\mu$ ,  $Z_\mu$  and  $Z'_\mu$ , we note that electric charge  $Q$  is given by

$$Q = I_{3L} + I_{3R} + I_{3Y} \quad (10)$$

$$\begin{array}{ccc} g & g & g_Y \\ W_{L\mu}^0 & W_{R\mu}^0 & Y_\mu^0 \end{array}$$

Below we define the gauge bosons and the couplings in the mass eigenbases,

$$\begin{aligned}
\frac{A_\mu}{e} &= \frac{W_{L\mu}^0}{g} + \frac{B_\mu}{g'}, & \frac{B_\mu}{g'} &= \frac{W_{R\mu}^0}{g} + \frac{Y_\mu^0}{g_Y}, & \frac{Z'_\mu}{g'} &= \frac{W_{R\mu}^0}{g_Y} - \frac{Y_{0\mu}}{g}, \\
\frac{1}{e^2} &= \frac{1}{g^2} + \frac{1}{g'^2}, & \frac{1}{g'^2} &= \frac{1}{g^2} + \frac{1}{g_Y^2}, \\
\frac{e}{g} &= \sin \theta_W, & \frac{e}{g'} &= \cos \theta_W, & g_Y &= \frac{g \tan \theta_W}{\sqrt{1 - \tan^2 \theta_W}}. \quad (11)
\end{aligned}$$

From the above definitions, one can obtain  $W_{L\mu}^0$ ,  $W_{R\mu}^0$  and  $Y_\mu^0$  in terms of the physical vector bosons  $A_\mu$ ,  $Z_\mu$  and  $Z'_\mu$  as:

$$\begin{aligned}
gW_{L\mu}^0 &= eA_\mu + g \cos \theta_W Z_\mu \\
gW_{L\mu}^0 - g_Y Y_{0\mu} &= \frac{g}{\cos \theta_W} Z_\mu + g \frac{\tan^2 \theta_W}{\sqrt{1 - \tan^2 \theta_W}} Z'_\mu \\
gW_{R\mu}^0 &= eA_\mu - g \frac{\sin^2 \theta_W}{\cos \theta_W} Z_\mu + g \sqrt{1 - \tan^2 \theta_W} Z'_\mu \\
g_Y Y_\mu^0 &= eA_\mu - g \frac{\sin^2 \theta_W}{\cos \theta_W} Z_\mu - g \frac{\tan^2 \theta_W}{\sqrt{1 - \tan^2 \theta_W}} Z'_\mu \\
gW_{R\mu}^0 - g_Y Y_\mu^0 &= \frac{g}{\sqrt{1 - \tan^2 \theta_W}} Z'_\mu \\
g(W_{L\mu}^0 - W_{R\mu}^0) &= \frac{g}{\cos \theta_W} Z_\mu - g \sqrt{1 - \tan^2 \theta_W} Z'_\mu \quad (12)
\end{aligned}$$

Using above relations, the neutral current interaction Lagrangian is given by

$$\begin{aligned}
\mathcal{L}_{\text{int}}^{\text{neutral}} &= \frac{-1}{4} \left\{ e \left[ -4\bar{e}_n \gamma^\mu e_n \right] A_\mu + g \left[ \bar{\nu}_n \gamma^\mu (1 - \gamma^5) - \bar{e}_n \gamma^\mu (1 - \gamma^5) e_n \right. \right. \\
&\quad \left. \left. - 4 \sin^2 \theta_W (-e_n \bar{\gamma}^\mu e_n) \right] \frac{Z_\mu}{\cos \theta_W} + g \left[ (\bar{N}_n \gamma^\mu (1 + \gamma^5) N_n - \bar{e}_n \gamma^\mu (1 + \gamma^5) e_n) \right. \right. \\
&\quad \left. \left. + \tan^2 \theta_W (\bar{\nu}_n \gamma^\mu (1 - \gamma^5) \nu_n - \bar{e}_n \gamma^\mu (1 - \gamma^5) e_n + 4\bar{e}_n \gamma^\mu e_n) \right] \frac{Z'_\mu}{\sqrt{1 - \tan^2 \theta_W}} \right\} \quad (13)
\end{aligned}$$

We conclude from Eq. (8) and Eq. (13), that except for lepton number violating term coupled to  $Y_\mu^\pm$ , we get exactly the same result for the lepton sector as those given by the left-right symmetric gauge group  $SU(2)_L \times SU(2)_R \times U(1)_{Y_1}$ , see for instance [4].

### III. SPONTANEOUS BREAKING OF THE GAUGE GROUP $SU(2)_L \times SU(2)_R \times SU(2)_Y$

In the first stage, the group  $SU(2)_Y$  is spontaneously broken to  $U(1)_{Y_1}$ , where  $Y_1$  is the hypercharge, by a scalar multiplet  $\Sigma$  which belong to singlet representation of  $SU(2)_L$ ,  $SU(2)_R$  and to

triplet representation of  $SU(2)_Y$ , i.e.  $\Sigma = (1, 1, 3)$  and can be written in the following form

$$\Sigma = \begin{pmatrix} H_\Sigma^+ \\ v_\Sigma + H_\Sigma^0 \\ H_\Sigma^- \end{pmatrix}, \quad (14)$$

where  $\langle \Sigma \rangle \equiv (0, v_\Sigma, 0)^T$  is the vacuum expectation value (vev) of  $\vec{\Sigma}$ . The mass term is given by

$$\begin{aligned} \mathcal{L}_{\text{mass}}^{Y^\pm} &= -\frac{1}{4}g_Y^2 \left[ (\langle \vec{\Sigma} \rangle \cdot \langle \vec{\Sigma} \rangle) (\vec{Y}^\mu \cdot \vec{Y}_\mu) - (\langle \vec{\Sigma} \rangle \cdot \vec{Y}^\mu) (\langle \vec{\Sigma} \rangle \cdot \vec{Y}_\mu) \right] \\ &= -\frac{1}{4}g_Y^2 V^2 [2Y^{+\mu}Y_\mu^-], \end{aligned} \quad (15)$$

where the mass of the gauge bosons  $Y^\pm$  is given by

$$m_{Y^\pm}^2 = \frac{1}{4}g_Y^2(2v_\Sigma^2) = \frac{1}{4}g^2 \frac{\tan^2 \theta_W}{1 - \tan^2 \theta_W} (2v_\Sigma^2). \quad (16)$$

The would be Goldstone bosons  $H_\Sigma^\pm$  have been absorbed in  $Y_\mu^\pm$  to give them longitudinal components and masses. The vector bosons  $Y_\mu^\pm$  are decoupled with one heavy Higgs scalar  $H_\Sigma^0$  and the electroweak unification group  $SU(2)_L \times SU(2)_R \times SU(2)_Y$  is broken to  $SU(2)_L \times SU(2)_R \times U(1)_{Y_1}$ . We are left with seven massless vector bosons  $(W_{L\mu}^\pm, W_{L\mu}^0)$ ,  $(W_{R\mu}^\pm, W_{R\mu}^0)$  and a singlet  $Y_\mu^0$  belonging to  $SU(2)_L$ ,  $SU(2)_R$  and  $U(1)_{Y_1}$ , respectively and the two doublets

$$\begin{pmatrix} \nu_n \\ e_n \end{pmatrix}_L, \quad \begin{pmatrix} e_n^c \\ -N_n^c \end{pmatrix}_L \quad (17)$$

belonging to representation 2 of  $SU(2)_L$  and  $SU(2)_R$  with hypercharge  $Y_1 = -L = \mp 1$ . The singlet vector boson  $Y_\mu^0 = B_{1\mu}$ ,  $g_Y = g_1$ .

In the second stage, the gauge group  $SU(2)_L \times SU(2)_R \times U(1)_{Y_1}$  (left-right symmetric group) is spontaneously broken to  $U(1)_{em}$  by three sets of scalars [4]:

$$\Delta_R : (1, 2, 2) = \begin{pmatrix} \eta^+ & \eta^{++} \\ \eta^\sigma & -\eta^+ \end{pmatrix}, \quad \langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ v_R/2 & 0 \end{pmatrix} \quad (18)$$

$$\Delta_L : (2, 1, 2) = \begin{pmatrix} \chi^+ & \chi^{++} \\ \chi^0 & -\chi^+ \end{pmatrix}, \quad \langle \Delta_L \rangle = \begin{pmatrix} 0 & 0 \\ v'_L/\sqrt{2} & 0 \end{pmatrix} \approx 0 \quad (19)$$

$$\phi : (2, 2, 0) = \begin{pmatrix} \phi^0 & \phi^+ \\ \phi^- & -\phi^0 \end{pmatrix}, \quad \langle \phi \rangle = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' \end{pmatrix} \quad (20)$$

The multiplet  $\Delta_R$ , generates the mass terms for the  $W_R^\pm$  and  $Z'$  gauge bosons by using Eq (12) as

$$\begin{aligned} \mathcal{L}_{\text{mass}}^{W_R^\pm, Z'} &= -\frac{1}{8}v_R^2 \left[ 2g^2 W_R^{+\mu} W_{R\mu}^- + 2 \left( gW_R^{0\mu} - gB_1^\mu \right) (gW_{R\mu}^0 - gB_{1\mu}) \right] \\ &= -\frac{1}{8}g^2 v_R^2 \left[ 2W_R^{+\mu} W_{R\mu}^- + \frac{2}{1 - \tan^2 \theta_W} Z'^\mu Z'_\mu \right]. \end{aligned} \quad (21)$$

Hence

$$m_{W_R^\pm}^2 = \frac{1}{4}g^2v_R^2, \quad m_{Z'}^2 = \frac{1}{4}\frac{2g^2v_R^2}{1 - \tan^2\theta_W} \quad (22)$$

At this stage the left-right symmetric group is broken to  $SU(2)_L \times U(1)_Y$ . The scalar multiplet  $\phi$  breaking this group to  $U(1)_{em}$ . The multiplet  $\phi$  generate the mass term for the  $W_L^\pm$  and  $Z$  as,

$$\begin{aligned} \mathcal{L}_{\text{mass}}^{W_L^\pm, Z} &= \frac{1}{4}g^2(\kappa^2 + \kappa'^2) \left[ \left( 2W_L^{+\mu}W_{L\mu}^- + 2W_R^{+\mu}W_{R\mu}^- \right) + \left( W_L^{0\mu} - W_R^{0\mu} \right) (W_{L\mu}^0 - W_{R\mu}^0) \right] \\ &\quad - g^2\kappa\kappa' \left( W_L^{+\mu}W_{R\mu}^- + W_R^{+\mu}W_{L\mu}^- \right) \end{aligned} \quad (23)$$

$$\begin{aligned} &= \frac{1}{4}g^2\kappa^2 \left[ 2W_L^{+\mu}W_{L\mu}^- + 2W_R^{+\mu}W_{R\mu}^- + \frac{1}{\cos^2\theta_W}Z^\mu Z_\mu - \frac{\sqrt{1 - \tan^2\theta_W}}{\cos\theta_W} (Z'^\mu Z_\mu + Z^\mu Z'_\mu) \right. \\ &\quad \left. + (1 - \tan^2\theta_W) Z'^\mu Z'_\mu \right] \end{aligned} \quad (24)$$

where in the last step we used Eq. (12) and the fact that  $\kappa' \ll \kappa$  (which one can select). Hence with  $\kappa' \ll \kappa \ll v_R$ , and  $\kappa = v_L/\sqrt{2}$ , we get

$$m_{W_L^\pm}^2 = \frac{1}{4}g^2v_L^2, \quad m_Z^2 = \frac{1}{4}\frac{g^2}{\cos^2\theta_W}v_L^2 \quad (25)$$

The scalar multiplet  $\Delta_R$  gives the Majorana mass term to the right-hand neutrino  $N_n$ :

$$\begin{aligned} \mathcal{L}_{\text{mass}}^{\text{Majorana}} &= - \left( e_n^T, -N_n^{cT} \right)_L C^{-1} i\tau_2 \langle \Delta_R \rangle^\dagger \begin{pmatrix} e_n \\ -N_n^c \end{pmatrix}_L + \text{h.c.} \\ &= - \left( e_n^T, -N_n^{cT} \right)_L C^{-1} \begin{pmatrix} 0 & 0 \\ 0 & v_R/2 \end{pmatrix} \begin{pmatrix} e_n \\ -N_n^c \end{pmatrix}_L + \text{h.c.} \\ &= -\frac{v_R}{2} \left[ N_{nL}^{cT} C^{-1} N_{nL}^c - \bar{N}_{nL}^c C \bar{N}_{nL}^{cT} \right] \\ &= -\frac{v_R}{2} \left[ N_{nR}^T C^{-1} N_{nR} + \text{h.c.} \right]. \end{aligned} \quad (26)$$

The multiplet  $\phi$  generates Dirac masses for the leptons. The Dirac mass term for the leptons is

$$\begin{aligned} \mathcal{L}_{\text{mass}}^{\text{Dirac}} &= - \left[ h_{1l_n} (\nu_n^T, e_n^+) C^{-1} \langle \phi \rangle i\tau_2 \begin{pmatrix} e_n^c \\ -N_n^c \end{pmatrix}_L + h_{2l_n} (\nu_n^T, e_n^+) C^{-1} i\tau_2 \langle \phi \rangle \begin{pmatrix} e_n^c \\ -N_n^c \end{pmatrix}_L \right] + \text{h.c.} \\ &= - \left[ -h_{1l_n} (\kappa\nu_{nL}^T C^{-1} N_{nL}^c + h_1\kappa' e_{nL}^T C^{-1} e_{nL}^c) - h_{2l_n} (\kappa'\nu_{nL}^T C^{-1} N_{nL}^c + \kappa e_{nL}^T C^{-1} e_{nL}^c) \right] + \text{h.c.} \\ &= - \left[ (h_{1l_n}\kappa + h_{2l_n}\kappa') \bar{N}_{nR}\nu_{nL} + (h_{1l_n}\kappa' + h_{2l_n}\kappa) (\bar{e}_{nR}e_{nL}) \right] + \text{h.c.} \\ &= - \left[ h_{1l_n}\kappa (\bar{N}_{nR}\nu_{nL} + \text{h.c.}) + h_{2l_n}\kappa (\bar{e}_{nR}e_{nL} + \text{h.c.}) \right] \\ &= -\frac{v_L}{\sqrt{2}} \left[ h_{1l_n} (\bar{\nu}_{nL}N_{nR} + \text{h.c.}) + h_{2l_n} (\bar{e}_{nL}e_{nR} + \text{h.c.}) \right], \end{aligned} \quad (27)$$

above in the second-last line above we used the approximation  $\kappa' \ll \kappa \ll v_R$ . On diagonalization, it gives Majorana mass terms  $m_{\nu_{Ln}} = \frac{m_{In}^2}{4M_{N_{Rn}}}$ . Moreover, the multiplet  $\phi$  also generate the quark masses, the mass term for quarks:

$$\mathcal{L}_{\text{mass}}^{\text{quark}} = -\frac{v_L}{\sqrt{2}} [h_{1q_n} (\bar{u}_{nL} u_{nR} + h.c.) + h_{2q_n} (\bar{d}_{nL} d_{nR} + h.c.)] \quad (28)$$

We end this section with the following remark. The content of the gauge vector bosons and breaking of the gauge group  $SU(2)_L \times SU(2)_R \times U(1)_{Y_1}$  by the Higgs scalar multiplets are characteristic of the gauge groups and are independent of the fermionic content of the model. In the left-right symmetric gauge group  $SU(2)_L \times SU(2)_R \times U(1)_{Y_1}$ , with  $Y_1 = B - L$  considered in Ref. [4] the left-handed quarks and antiquarks doublets

$$\begin{pmatrix} u_n \\ d'_n \end{pmatrix}_L, \quad \begin{pmatrix} d_n^c \\ -u_n^c \end{pmatrix}_L$$

have  $Y_1 = B$ ,  $Y_1 = \pm \frac{1}{3}$  whereas lepton (antilepton) multiplets have  $Y_1 = -L$ ,  $Y_1 = \mp 1$  as given in Eq. (17). The vector bosons  $W_{L\mu}^\pm$  and  $W_{R\mu}^\pm$  are coupled to leptons and quarks with no difference, where as the vector boson  $B_{1\mu}$  associated with  $U(1)_{Y_1}$  is coupled to leptons with  $Y_1 = -L$  and to quarks with  $Y_1 = B$ . The coupling of Higgs scalar with  $Y_1 = 0$  and  $Y_1 = 2$  are coupled to all the fermions of the group.

#### IV. EFFECTIVE LAGRANGIAN FOR CHARGED LEPTON NUMBER VIOLATING DECAYS

In the Standard Model (SM) charged lepton ( $\mu$  and  $\tau$ ) decays are mediated by the vector bosons  $W_{L\mu}^\pm$ . From the first term of Eq. (8), the effective Lagrangian for these decays is given by

$$\mathcal{L}_{\text{eff}}^{\text{SM}} = \frac{G_F}{\sqrt{2}} [\bar{\nu}_n \gamma^\mu (1 - \gamma^5) e_n] [\bar{e}_m \gamma_\mu (1 - \gamma^5) \nu_m], \quad (29)$$

where  $G_F = \sqrt{2}g^2/8m_W^2$ , is the Fermi constant. After Fieriez reordering

$$\mathcal{L}_{\text{eff}}^{\text{SM}} = \frac{G_F}{\sqrt{2}} [\bar{e}_m \gamma^\mu (1 - \gamma^5) e_n] [\bar{\nu}_n \gamma_\mu (1 - \gamma^5) \nu_m]. \quad (30)$$

In particular, for  $\mu^- \rightarrow \nu_\mu + e^- + \bar{\nu}_e$ :

$$\mathcal{L}_{\text{eff}}^{\text{SM}}(\mu\text{-decay}) = \frac{G_F}{\sqrt{2}} [\bar{e} \gamma^\mu (1 - \gamma^5) \mu] [\bar{\nu}_\mu \gamma_\mu (1 - \gamma^5) \nu_e]. \quad (31)$$

The effective Lagrangian for the charged lepton flavor violating (LFV) decays mediated by  $Y_\mu^\pm$ :

$$\mathcal{L}_{\text{eff}}^{\text{LFV}} = \frac{G_Y}{\sqrt{2}} [\bar{e}_m^c \gamma^\mu (1 - \gamma^5) \nu_n] [\bar{\nu}_{n'} \gamma_\mu (1 - \gamma^5) e_{m'}^c], \quad (32)$$

with

$$\frac{G_Y}{\sqrt{2}} = \frac{g_Y^2}{8m_Y^2} = \frac{g^2}{8} \frac{\tan^2 \theta_W}{1 - \tan^2 \theta_W} \frac{1}{m_Y^2}.$$

Fierz reordering gives

$$\begin{aligned} \mathcal{L}_{\text{eff}}^{\text{LFV}} &= \frac{G_Y}{\sqrt{2}} [\bar{e}_m^c \gamma^\mu (1 - \gamma^5) e_{m'}^c] [\bar{\nu}_{n'} \gamma_\mu (1 - \gamma_5) \nu_n] \\ &= -\frac{G_Y}{\sqrt{2}} [e_m^T C^{-1} \gamma^\mu (1 - \gamma^5) C \bar{e}_{m'}^T] [\bar{\nu}_{n'} \gamma_\mu (1 - \gamma_5) \nu_n] \\ &= -\frac{G_Y}{\sqrt{2}} [\bar{e}_{m'} \gamma^\mu (1 + \gamma^5) e_m] [\bar{\nu}_{n'} \gamma_\mu (1 - \gamma_5) \nu_n]. \end{aligned} \quad (33)$$

There is a choice of the possible assignments of three generations of leptons. The natural assignment is as follow:

$$(i) \quad D(e) : D(123) = \begin{pmatrix} \nu_e & e^+ \\ e^- & -N_e^c \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\mu & \mu^+ \\ \mu^- & -N_\mu^c \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\tau & \tau^+ \\ \tau^- & -N_\tau^c \end{pmatrix}_L \quad (34)$$

For the assignment (i), the  $\mu$  and  $\tau$  decays are as follows

$$\mu^- \rightarrow \bar{\nu}_\mu + e^- + \nu_e, \quad \Delta L_\mu = -2, \quad \Delta L_e = 2, \quad \nu_e \leftrightarrow \nu_\mu, \quad (35)$$

$$\tau^- \rightarrow \bar{\nu}_\tau + \mu^- + \nu_\mu, \quad \Delta L_\tau = -2, \quad \Delta L_\mu = 2, \quad \nu_\mu \leftrightarrow \nu_\tau, \quad (36)$$

$$\tau^- \rightarrow \bar{\nu}_\tau + e^- + \nu_e, \quad \Delta L_\tau = -2, \quad \Delta L_e = 2, \quad \nu_e \leftrightarrow \nu_\tau. \quad (37)$$

These decays stimulate the neutrino oscillations  $\nu_e \rightarrow \nu_\mu$  in the  $\mu$ -decay and  $\nu_\mu \rightarrow \nu_\tau$ ,  $\nu_\tau \rightarrow \nu_e$  in the  $\tau$ -decay.

The lepton flavor violating effective Lagrangian for  $\mu^- \rightarrow \bar{\nu}_\mu + e^- + \nu_e$  can be written as

$$\mathcal{L}_{\text{eff}}^{\text{LFV}}(\mu\text{-decay}) = -\frac{G_Y}{\sqrt{2}} [\bar{e} \gamma^\mu (1 + \gamma^5) \mu] [\bar{\nu}_e \gamma_\mu (1 - \gamma_5) \nu_\mu] \quad (38)$$

and similarly for the  $\tau$ -decays, replace  $\mu \rightarrow \tau$ ,  $\nu_\mu \rightarrow \nu_\tau$ , and  $\nu_e \rightarrow \nu_\mu$ , in the above expression.

Using the permutation  $D(123) \rightarrow D(231)$ :

$$(ii) \quad D(231) = \begin{pmatrix} \nu_e & \mu^+ \\ e^- & -N_\mu^c \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\mu & \tau^+ \\ \mu^- & -N_\tau^c \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\tau & e^+ \\ \tau^- & -N_e^c \end{pmatrix}_L \quad (39)$$

For the assignment (ii), the lepton flavor violating decays are

$$\mu^- \rightarrow \bar{\nu}_e + e^- + \nu_\tau, \quad \Delta L_\mu = -1, \quad \Delta L_\tau = 1, \quad \nu_\mu \rightarrow \nu_\tau \quad (40)$$

$$\tau^- \rightarrow \bar{\nu}_\mu + \mu^- + \nu_e, \quad \Delta L_\tau = -1, \quad \Delta L_e = 1, \quad \nu_\tau \rightarrow \nu_e \quad (41)$$

$$\tau^- \rightarrow \bar{\nu}_\mu + e^- + \nu_\tau, \quad \Delta L_\mu = -1, \quad \Delta L_e = 1, \quad \nu_e \rightarrow \nu_\mu \quad (42)$$



These decays also stimulate the neutrino oscillations  $\nu_\mu \rightarrow \nu_\tau$ ,  $\nu_\tau \rightarrow \nu_e$ ,  $\nu_e \rightarrow \nu_\mu$  in  $\mu$  and  $\tau$  decays. In this assignment (ii), the charged LFV effective Lagrangian for the decay  $\mu^- \rightarrow \bar{\nu}_e + e^- + \nu_\tau$  is

$$\mathcal{L}_{\text{eff}}^{\text{LFV}}(\mu\text{-decay}) = -\frac{G_Y}{\sqrt{2}}[\bar{e}\gamma^\mu(1+\gamma^5)\mu][\bar{\nu}_\tau\gamma_\mu(1-\gamma_5)\nu_e]. \quad (43)$$

The Feynman amplitude for the  $\mu$  decay in the Standard Model is given by [c.f. Eq. (30)] [1]

$$|M_{\mu\text{-decay}}^{\text{SM}}|^2 = \sum_{\text{spin}} |F_{\mu\text{-decay}}^{\text{SM}}|^2 \sim \frac{G_F^2}{2} 4 p_2 \cdot k_1 p_1 \cdot k_2, \quad (44)$$

and for the lepton flavor violating  $\mu$  decay is given by [c.f. Eq. (38)]

$$|M_{\mu\text{-decay}}^{\text{LFV}}|^2 = \sum_{\text{spin}} |F_{\mu\text{-decay}}^{\text{LFV}}|^2 \sim \frac{G_Y^2}{2} 4 p_1 \cdot k_1 p_2 \cdot k_2, \quad (45)$$

where  $p_1$ ,  $p_2$ ,  $k_1$  and  $k_2$  are the 4-momenta of  $\mu$ ,  $e$ ,  $\nu_\mu$  and  $\nu_e$ . From Eq. (44), the decay width  $d\Gamma$ , for  $\mu^- \rightarrow \nu_\mu + e^- + \bar{\nu}_e$  is given by [1]

$$d\Gamma_{\mu\text{-decay}}^{\text{SM}} = \frac{G_F^2}{12\pi^3} m_\mu p_e dE_e [3W E_e - 2E_e^2 - m_e^2], \quad \text{where} \quad W \equiv \frac{m_\mu^2 + m_e^2}{2m_\mu} \quad (46)$$

After integration we get,

$$\Gamma_{\mu\text{-decay}}^{\text{SM}} = \tau_\mu^{-1} = \frac{G_F^2}{192\pi^3} m_\mu^5 [1 - \frac{8m_e^2}{m_\mu^2}]. \quad (47)$$

From Eq. (45), we get exactly the same expressions for the LFV case as those given in Eq. (46) and Eq. (47) with  $G_F^2 \rightarrow G_Y^2$ . For  $\tau$  decays, replace  $m_\mu \rightarrow m_\tau$ ,  $m_e \rightarrow m_\mu$  for  $\tau \rightarrow \mu$  and for  $\tau \rightarrow e$ ,  $m_\mu \rightarrow m_\tau$ . Hence, for the assignment (i)

$$\mathcal{R}_{\mu\text{-decay}}^{\text{LFV}} \equiv \frac{\Gamma_{\mu\text{-decay}}^{\text{LFV}}(\mu^- \rightarrow \bar{\nu}_\mu + e^- + \nu_e)}{\Gamma_{\mu\text{-decay}}^{\text{SM}}(\mu^- \rightarrow \nu_\mu + e^- + \bar{\nu}_e)} = \frac{G_Y^2}{G_F^2} = \frac{g_Y^4}{g^4} \left( \frac{m_{W_L}}{m_Y} \right)^4 = \left( \frac{\sin^2 \theta_W}{1 - 2\sin^2 \theta_W} \right)^2 \left( \frac{m_{W_L}}{m_Y} \right)^4,$$

Moreover, for the  $\tau$ -decay we get the same ratio, i.e.  $\mathcal{R}_{\tau\text{-decay}}^{\text{LFV}} = \mathcal{R}_{\mu\text{-decay}}^{\text{LFV}}$ . Similar expressions for the decays  $\mu^- \rightarrow \bar{\nu}_e + e^- + \nu_\tau$ ,  $\tau^- \rightarrow \bar{\nu}_\mu + \mu^- + \nu_e$  and  $\tau^- \rightarrow \bar{\nu}_\mu + \mu^- + \nu_e$  for the assignment (ii). The probability to observe lepton flavor violating  $\mu$  and  $\tau$  decays must be less than  $10^{-6}$ , since the SM decay rate for  $\mu$  decay is in agreement with the experimental value up to six places of decimals. For example, using  $\sin^2 \theta_W = 0.23$ ,  $m_{W_L} \approx 80.38$  GeV, the probability to observe lepton flavor violating decay is  $2.9 \times 10^{-8}$  for  $m_Y = 50m_{W_L} \approx 4$  TeV and  $1.5 \times 10^{-9}$  for  $m_Y = 100m_{W_L} \approx 8$  TeV and for  $m_Y = 65$  TeV the probability is  $5.0 \times 10^{-13}$ .

The energy spectrum given in Eq. (46) is modified by  $\mathcal{L}_{\text{eff}}^{\text{LFV}}$  for flavor violating decays. The Feynman amplitude for  $\mu$  decay into electron is given by

$$\begin{aligned} F_{\mu\text{-decay}}^{\text{SM}} &= \frac{G_F}{\sqrt{2}} [\bar{u}(p_2)\gamma^\mu(1-\gamma^5)u(p_1)] [\bar{u}(k_1)\gamma_\mu(1-\gamma_5)v(k_2)] \\ F_{\mu\text{-decay}}^{\text{LFV}} &= -\frac{G_Y}{\sqrt{2}} [\bar{u}(p_2)\gamma^\mu(1+\gamma^5)u(p_1)] [\bar{u}(k_2)\gamma_\mu(1-\gamma_5)v(k_1)], \end{aligned} \quad (48)$$

which leads to

$$|M_{\mu\text{-decay}}|^2 \sim G_F^2 4 p_2 \cdot k_1 p_1 \cdot k_2 - 2G_F G_Y (-4m_\mu m_e k_1 \cdot k_2) \quad (49)$$

From Eq. (49), the modified energy spectrum is given by

$$d\Gamma = \frac{G_F^2}{12\pi^3} m_\mu p_e dE_e [3W E_e - 2E_e^2 - m_e^2 - 12(\frac{\sin^2 \theta_W}{1 - \sin^2 \theta_W}) \frac{m_{W_L}^2}{m_Y^2} m_e \times (W - E_e)]. \quad (50)$$

## V. CONCLUSIONS

The electroweak unification group  $SU(2)_L \times SU(2)_R \times SU(2)_Y$  is broken to  $SU(2)_L \times SU(2)_R \times U(1)_{Y_1}$  by a scalar multiple  $\Sigma$  that belongs to the triplet representation of  $SU(2)_Y$  and singlet of  $SU(2)_L \times SU(2)_R$ , the vector boson  $Y^\pm$  acquired mass  $m_{Y^\pm}^2 = \frac{1}{2} \frac{\tan^2 \theta_W}{1 - \tan^2 \theta_W} g^2 v_\Sigma^2$ , where  $v_\Sigma^2 \gg v_R^2 \gg v_L^2$ . The vector bosons  $Y^\pm$  are decoupled, with one heavy Higgs scalar  $H_\Sigma^0$  and the residual group  $SU(2)_L \times SU(2)_R \times U(1)_{Y_1}$  where  $Y_1$  is the hypercharge has all the features of the left-right symmetric electroweak unification group  $SU(2)_L \times SU(2)_R \times U(1)_{Y_1}$  with hypercharge  $Y_1 = B - L$ .  $Y_1 = B = \pm \frac{1}{3}$  for the quark multiplets and  $Y_1 = -L = \mp 1$  for the lepton multiplets. Addition of quark multiplet does not change any other feature of the group. The probability to observe charged lepton flavor violating decays is  $\leq 10^{-9}$ .

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